On Two-Phase Spherical Motion Generation

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ABSTRACT

This work presents a method to design four revolute spherical mechanisms to approximate two phases of prescribed rigid-body positions. Using this method, four revolute spherical motion generators can be designed to approximate two phases of prescribed rigid-body positions using the same mechanism hardware. This work considers moving-pivot adjustable four revolute spherical motion generators with constant crank and follower lengths for two-phase motion generation.

INTRODUCTION

Conventional and Two-Phase Motion Generation

Motion generation involves the quantitative or qualitative design of a mechanism to achieve a series of prescribed rigid-body positions. For the planar four-bar loading mechanism illustrated in, Fig. 1, the rigid-body is the carrying block and the rigid-body positions in space is represented by coordinates of variables \( p, q \) and \( r \). Multi-phase motion generation involves the quantitative or qualitative design of an adjustable mechanism to achieve multiple series of prescribed rigid-body positions. In multi-phase motion generation, the user can design a single mechanism (or motion generator) to achieve multiple series (also called "phases") of prescribed rigid-body positions using essentially the same mechanism hardware. For example, if rigid-body positions 1, 2 and 3 in Fig. 2 are achieved when the four-bar mechanism incorporates the moving pivots \( a_1 \) and \( b_1 \), and positions 1, 4 and 5 are achieved when moving pivots \( a_{1n} \) and \( b_{1n} \) are incorporated, this mechanism could be classified as a two-phase adjustable planar motion generator.

![Fig. 1. Loading mechanism on a conveyor belt line.](image1)

![Fig. 2. Two phase adjustable planar motion generator and rigid-body positions achieved.](image2)
Four Revolute Spherical Mechanisms and Applications

Due to the link geometry and joint axis orientation of the four revolute spherical mechanism (Fig. 3), its workspace is spherical as opposed to the planar workspaces produced by planar mechanisms. The capacity for spherical manipulation of rigid-bodies in a mechanism has been proven to be quite advantageous when the rigid-body is a camera (Fig. 4, http://wwwrobot.gmc.ulaval.ca/recherche/theme01_a.html, 2006) or a fan (Fig. 5, http://dionne.me.coe.fit.edu/~rassl/research.html#fan, 2006). In Fig. 4, a camera is mounted on a 3-DOF spherical mechanism. This mechanism is called the “Agile Eye” and its responsiveness is comparable to that of the human eye. In Fig. 5, a fan is mounted to a four revolute spherical mechanism. This mechanism is called “The Infinity Fan” and is capable of thoroughly circulating air in a room even when facing a corner. If one can design such spherical mechanisms with adjustable features (for example, adjustable moving pivots) a single “Agile Eye,” “Infinity Fan” or any other spherical mechanism could be able to achieve additional rigid-body orientations while incorporating the same mechanism hardware. As a result, a single mechanical system could have a greater degree of flexibility and usefulness for multiple (but distinct) applications.

![Fig. 3. The four revolute spherical mechanism.](image3)

![Fig. 4. A 3-DOF spherical manipulator (the “Agile Eye”).](image4)

Literature Review and Scope of Work

The number of methods (Ahmad, 1979; Beaudrot, 1969; Bonnell, 1966; Chang, 2001; Chuenchom, 1997; Funabshi, 1986; McGovern, 1973; Shimojima, 1983; Tao, 1978, 1979; Wang, 1996; Wilhelm, 1989) available for the design of spherical mechanisms is modest compared to those for planar mechanisms. Sodhi and Shoup (1982, 1984) and Sodhi, Wilhelm and Shoup (1985) presented works that included the analysis of the axodes of the four revolute spherical mechanism and the synthesis of these mechanisms via Instant Screw Axes (ISAs) and curve matching. Gilmartin and Duffy (1972) examined type and mobility analysis of the four revolute spherical mechanism. Tong and Chang (1992) presented the synthesis of four revolute spherical mechanisms via the pole method. McCarthy and Boduluri (1992, 2000) considered the generalization of planar rectification theory to four revolute spherical mechanisms as well as an approach to the finite position synthesis of these mechanisms that unites traditional precision theory with recent results in approximate position synthesis. Ruth and McCarthy (1999) described a computer-aided design software system for four revolute spherical mechanisms that is based on Burmester’s Theory.

A method to design four revolute spherical mechanisms to approximate two phases of prescribed rigid-body positions is presented in this paper. Using this method, four revolute spherical motion generators are designed to approximate two phases of prescribed rigid-body positions using the same mechanism hardware. This work considers moving-pivot adjustable four revolute spherical motion generators with constant crank and follower lengths for two-phase motion generation.
SPATIAL RIGID-BODY GUIDANCE

Fig. 6. Four revolute spherical motion generator with rigid body variables p, q, and r.

Figure 6 illustrates a four revolute spherical motion generator. In this work, link a$_0$-a$_1$ will be the designated crank link and link b$_0$-b$_1$ is the designated follower link. Links a$_0$-a$_1$ and b$_0$-b$_1$ of the mechanism must satisfy two conditions. One condition is that the chord length between the fixed and moving pivots of both links must remain constant throughout each phase. The other condition is that the fixed and moving pivots must all lie on a sphere of unit radius. Given a fixed pivot a$_0$ and a moving pivot a$_1$, the constant chord length constraint and the unit sphere constraints (Sandor, 1984; Suh, 1978) are expressed in Eqns. (1) through (3).

$$(a_j - a_0)^T(a_j - a_0) = (a_j - a_0)^T(a_j - a_0), \ j = 2, 3, \ldots, n, \quad (1)$$

$$(a_0)^T(a_0) = 1, \quad (2)$$

$$(a_1)^T(a_1) = 1, \quad (3)$$

where, a$_0$ = (a$_{0x}$, a$_{0y}$, a$_{0z}$, 1), a$_1$ = (a$_{1x}$, a$_{1y}$, a$_{1z}$, 1), a$_j$ = [D$_j$]a$_1$.

and

$$[D_j] = \begin{bmatrix}
    p_{jx} & q_{jx} & r_{jx} & 0 & 0 & 0 & 0 & 0
    p_{jy} & q_{jy} & r_{jy} & 0 & 0 & 0 & 0 & 0
    p_{jz} & q_{jz} & r_{jz} & 0 & 0 & 0 & 0 & 0
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Equation 1 is rewritten as Eqn. (5). In Eqn. (5), the variable L represents the length of the chord connecting the fixed and moving pivots.

$$(a_j - a_0)^T(a_j - a_0) = L^2 \quad j = 2, 3, \ldots, n. \quad (5)$$

Variables p, q, and r in Eqn. (4) represent the spatial position of the rigid-body (the coupler in Fig. 6).

TWO-PHASE SPHERICAL MOTION GENERATION EQUATIONS

This work considers two-phase moving pivot adjustments of the four revolute spherical mechanism (with fixed crank and follower lengths). For such a problem, the required unknowns are a$_0$, a$_1$, a$_{1n}$, b$_0$, b$_1$ and b$_{1n}$. Variables a$_1$ and b$_1$ represent the moving pivots required to achieve the prescribed phase 1 rigid-body positions. Variables a$_{1n}$ and b$_{1n}$ represent the moving pivots required to achieve the prescribed phase 2 rigid-body positions. Variables a$_0$ and b$_0$ represent the fixed pivots required to achieve the prescribed phase 1 and phase 2 rigid-body positions. Since variables a$_0$, a$_1$, and a$_{1n}$ have three unknown components each (the X, Y and Z-components) there are a total of 9 variables to determine (variables b$_0$, b$_1$ and b$_{1n}$ have three unknown components each also).

$$a_0 = (a_{0x}, a_{0y}, a_{0z}, 1), \ a_1 = (a_{1x}, a_{1y}, a_{1z}, 1),$$

$$a_{1n} = (a_{1nx}, a_{1ny}, a_{1nz}, 1),$$

$$b_0 = (b_{0x}, b_{0y}, b_{0z}, 1), \ b_1 = (b_{1x}, b_{1y}, b_{1z}, 1),$$

$$b_{1n} = (b_{1nx}, b_{1ny}, b_{1nz}, 1).$$

Equations (6) through (10) are used to calculate eight unknowns in a$_0$, a$_1$ and a$_{1n}$. The link dimension L$_j$ and variable a$_{0c}$ are specified. Equations (6) through (10) represent the phase 1 and phase 2 adjustments of link a$_c$-a.

$$(D_{ij}[a_1 - a_0])^T((D_{ij}[a_1 - a_0]) - L_j^2 = 0, \ j = 2, 3, 4 \quad (6)$$

$$(D_{3k}[a_{1n} - a_0])^T((D_{3k}[a_{1n} - a_0]) - L_k^2 = 0, \ k = 5, 6, 7 \quad (7)$$

$$(a_0)^T(a_0) = 1 \quad (8)$$

$$(a_1)^T(a_1) = 1 \quad (9)$$

$$(a_{1n})^T(a_{1n}) = 1 \quad (10)$$

Equations (11) through (15) are used to calculate eight unknowns in b$_0$, b$_1$ and b$_{1n}$. The link dimension L$_2$ and variable b$_{0c}$ are specified. Equations (11) through (15) represent the phase 1 and phase 2 adjustments of link b$_c$-b.

$$(D_{ij}[b_1 - b_0])^T((D_{ij}[b_1 - b_0]) - L_j^2 = 0, \ j = 2, 3, 4 \quad (11)$$

$$(D_{3k}[b_{1n} - b_0])^T((D_{3k}[b_{1n} - b_0]) - L_k^2 = 0, \ k = 5, 6, 7 \quad (12)$$

$$(b_0)^T(b_0) = 1 \quad (13)$$

$$(b_1)^T(b_1) = 1 \quad (14)$$

$$(b_{1n})^T(b_{1n}) = 1 \quad (15)$$

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MOVING PIVOT ADJUSTMENT EXAMPLE

This section demonstrates the synthesis of a two-phase moving pivot adjustable spherical motion generator with constant crank and follower lengths. Table 1 includes two phases of prescribed spherical rigid-body positions.

Equations (6) through (10) were used to calculate eight of the nine unknowns in \( a_0, a_1 \) and \( a_{1n} \). The nonlinear equation set is solved by Newton-Raphson method iteration algorithm, and programming by symbolic mathematics package, Mathematica. The iterations could rapidly converge to one set of solutions with reasonable initial guesses. Variables \( L_1 \) and \( a_{0c} \) were specified to 0.6840 and 0.7071 respectively. Using the following initial guesses:

\[
\begin{align*}
\ a_0 &= -0.5, a_0 &= 0, \\
\ a_1 &= a_{1n} &= (-0.5, 0.5, 0.5), \\
\end{align*}
\]

the adjustable crank link solutions converge to

\[
\begin{align*}
\ a_0 &= -0.7071, a_0 &= -0.0008, \\
\ a_1 &= (-0.5421, 0.6423, 0.5418),
\end{align*}
\]

Equations (11) through (15) were used to calculate eight of the nine unknowns in \( b_0, b_1 \) and \( b_{1n} \). Variables \( L_2 \) and \( b_{0c} \) were specified to 0.7654 and 0.7071 respectively. Using the following initial guesses:

\[
\begin{align*}
\ b_0 &= -0.5, b_0 &= 0, b_1 &= b_{1n} &= (0.5, 0.5, 0.5), \\
\end{align*}
\]

the adjustable follower link solutions converge to

\[
\begin{align*}
\ b_0 &= 0.7071, b_0 &= 0.0058, \\
\ b_1 &= (0.4982, 0.7110, 0.4963), \\
\ b_{1n} &= (0.3635, 0.6835, 0.6180).
\end{align*}
\]

The synthesized adjustable four revolute spherical motion generator is illustrated in Fig. 7. Table 2 includes the rigid-body points achieved by the synthesized mechanism. In Table 2, rigid-body positions 2, 3 and 4 were achieved with crank displacement angles of \(-10^\circ\), \(-20^\circ\) and \(-30^\circ\) respectively. Rigid-body positions 5 and 6 were achieved with crank displacement angles \(-10^\circ\) and \(-20^\circ\) respectively.

### Table 1. Prescribed rigid-body positions for the adjustable four revolute spherical motion generator.

<table>
<thead>
<tr>
<th>PHASE 1</th>
<th>p</th>
<th>q</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pos. 1</td>
<td>-0.2081, 0.7647, 0.3468</td>
<td>0, 0.8355, 0.2239</td>
<td>0.2271, 0.7601, 0.3448</td>
</tr>
<tr>
<td>Pos. 2</td>
<td>-0.1593, 0.7588, 0.3835</td>
<td>0.0297, 0.8322, 0.2341</td>
<td>0.2715, 0.7558, 0.3214</td>
</tr>
<tr>
<td>Pos. 3</td>
<td>-0.1151, 0.7419, 0.4296</td>
<td>0.0536, 0.8229, 0.2610</td>
<td>0.3047, 0.7458, 0.3149</td>
</tr>
<tr>
<td>Pos. 4</td>
<td>-0.0766, 0.7145, 0.4815</td>
<td>0.0713, 0.8080, 0.3004</td>
<td>0.3279, 0.7330, 0.3217</td>
</tr>
<tr>
<td>Pos. 5</td>
<td>-0.2081, 0.7647, 0.3468</td>
<td>0, 0.8355, 0.2239</td>
<td>0.2271, 0.7601, 0.3448</td>
</tr>
<tr>
<td>Pos. 6</td>
<td>-0.1886, 0.7506, 0.3863</td>
<td>-0.0083, 0.8335, 0.2312</td>
<td>0.2397, 0.7705, 0.3115</td>
</tr>
<tr>
<td>Pos. 7</td>
<td>-0.1716, 0.7249, 0.4397</td>
<td>-0.0200, 0.8234, 0.2642</td>
<td>0.2399, 0.7726, 0.3061</td>
</tr>
</tbody>
</table>

### Table 2. Rigid-body positions achieved by the synthesized four revolute spherical motion generator.

<table>
<thead>
<tr>
<th>PHASE 1</th>
<th>p</th>
<th>q</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pos. 1</td>
<td>-0.2081, 0.7647, 0.3468</td>
<td>0, 0.8355, 0.2239</td>
<td>0.2271, 0.7601, 0.3448</td>
</tr>
<tr>
<td>Pos. 2</td>
<td>-0.1593, 0.7589, 0.3835</td>
<td>0.0298, 0.8321, 0.2342</td>
<td>0.2715, 0.7557, 0.3216</td>
</tr>
<tr>
<td>Pos. 3</td>
<td>-0.1150, 0.7419, 0.4296</td>
<td>0.0538, 0.8229, 0.2611</td>
<td>0.3048, 0.7457, 0.3150</td>
</tr>
<tr>
<td>Pos. 4</td>
<td>-0.0765, 0.7146, 0.4814</td>
<td>0.0715, 0.8080, 0.3003</td>
<td>0.3280, 0.7329, 0.3217</td>
</tr>
<tr>
<td>Pos. 5</td>
<td>-0.2081, 0.7647, 0.3468</td>
<td>0, 0.8355, 0.2239</td>
<td>0.2271, 0.7601, 0.3448</td>
</tr>
<tr>
<td>Pos. 6</td>
<td>-0.1881, 0.7508, 0.3864</td>
<td>-0.0078, 0.8335, 0.2312</td>
<td>0.2401, 0.7704, 0.3114</td>
</tr>
<tr>
<td>Pos. 7</td>
<td>-0.1707, 0.7251, 0.4397</td>
<td>-0.0193, 0.8234, 0.2641</td>
<td>0.2406, 0.7726, 0.3057</td>
</tr>
</tbody>
</table>
DISCUSSION

In this work, the authors used the mathematical analysis software package MATHEMATICA to compute all of the numerical results displayed. This package can express a numerical value to four decimal places. The CAD package AutoCAD used to specify the rigid-body positions in the example problem provided. A CAD package that enables one to create and manipulate objects in three dimensional space can also enable the user to prescribe rigid body positions judiciously.

CONCLUSION

The two-phase spherical motion generation method presented in this work was demonstrated to be effective in synthesizing a moving pivot adjustable four revolute spherical motion generator to approximate two phases of prescribed rigid-body positions using the same mechanism hardware.

REFERENCES


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Sodhi, R.S. and Shoup, T.E., “Axodes for the Four


摘要

本研究提出一設計方法，使用 4R 球型機構，使得球型機構運動逼近兩個系列的剛體運動空間位置。使用這種設計方法，4R 球型機構可以在使用單一機構的硬體裝置下，完成逼近兩個系列的剛體運動空間位置的成果。本研究主要使用移動節點的可調整性，以及固定長度的曲柄與從動連桿，考慮雙相，可調式移動節點的 4R 球型機構。