Geometrical Design of Roller Drives with Two-Tooth Difference

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Keywords: Roller drives, geometrical design, speed reducer, CAD.

ABSTRACT

This paper presents the design procedures and algorithm to design a new type roller drive. The geometrical design is obtained by the coordinate transformation, kinematics and envelope theory. Four examples are presented and using CAD constructs the solid modeling to demonstrate the feasibility of this approach. This design model can be a useful reference as a design case for other tooth profiles.

INTRODUCTION

Speed reducers are used widely in various applications for speed and torque conversion purposes. The cycloid drive is more compact, higher speed reduction, and higher mechanical advantage than planetary gear trains in a single stage [Botsiber and Kingston, 1956]. For the two-tooth difference of the cycloid drives, Chang and Liu [2000] researched the cycloid drives with two-tooth difference on under cutting; Lyu and Lai [2004] studied the geometric design of the cycloid drives with conical meshing elements for two-tooth difference. The epicycloidal planet profile has cusps on the addendum, whose cusps are an inevitable drawback. With gearing contact, the cusps will cause noise and damage to the gearing elements. Lai and Lyu [2006] studied the addendum modifying of cycloid drives with two-tooth difference on the epicycloidal planet gear. Cycloid drives are widely used in industry, but both the cost and required manufacturing precision are very high. Here a speed reducer is introduced, the roller driver (Roladrive) that employs rollers instead of cut gears giving the advantages of easy manufacture and low cost. It was initially applied as a speed reduction mechanism in Ko [1991]. Yan and Lai [2000] initially studied the characteristics and mechanical efficiency of the roller drives. Later, Lai [2005, 2006] studied the geometrical design of a roller drive and geometry of a pinion with two circularly arrayed conical teeth for roller drives. Due to the conventional roller drives only has one-tooth difference between the ring gear and pinion, now we present a new type roller drives. This reducer has two-tooth difference between the ring gear and pinion teeth number.

The purpose of this work is proposed a new concept for roller drivers with two-tooth difference and presents the procedures and mathematical algorithm to realize the geometrical design. Then using CAD constructs the solid modeling to demonstrate the feasibility of this approach.

TOPOLOGICAL STRUCTURE AND MESHING EQUATIONS

The topological structure of the roller drives with two-tooth difference as shown in Figure 1. Member 1 is a frame; member 2 is a ring gear, whose members include the ring gear body, cylindrical meshing elements, and ring gear pins; member 3 is a pinion, whose members include the pinion body and cylindrical teeth; member 4 is a crank; member 5 is an output disk, whose disk pin is a floating connection with member 3.
According to the topological structure of Fig. 1, we define two fixed and three movable coordinate systems as shown in Fig. 2. Where the fixed coordinate system \((xyz)_f\) is rigidly connected to the frame, while the fixed coordinate system \((xyz)_b\) is also connected to the frame. Moving coordinate systems \((xyz)_2\), \((xyz)_r\), and \((xyz)_3\) are rigidly connected to the ring gear, the cylindrical meshing element, and the pinion, respectively. The ring gear rotates about the \(z_2\) axis and the pinion rotates about the \(z_3\) axis. Origins \(o_f\) and \(o_2\) are coincident and located at the centre of the ring gear, while origins \(o_b\) and \(o_3\) are coincident and located at the centre of the pinion. Origin \(o_r\) is coincident and located at the centre of the cylindrical meshing element. Axis \(x_f\) is parallel with axis \(x_b\). The directions of axes \(z_f\), \(z_b\), \(z_2\), \(z_3\), and \(z_r\) are perpendicular to the \(xy\) plane. Angles \(\phi_2\) and \(\phi_3\) are the angular displacements of the ring gear and pinion, respectively. Positive \(\phi_2\) and \(\phi_3\) are measured counterclockwise with respect to axis \(z_2\) and axis \(z_3\), respectively. The distance between the centres of the ring gear and the meshing element is denoted as \(d\). The distance between the axes of rotation of the ring gear and the pinion is \(e\). Symbols \(R_2\) and \(R_3\) are the radii of the centroids of the ring gear and the pinion, respectively.

According to above coordinate systems and parameters defined, we can apply the transformation matrices of the coordinate systems to obtain the following homogeneous coordinates matrices [Denavit & Hartenberg, 1955].

\[
M_{3, r} = \begin{bmatrix}
\cos(\phi_2 - \phi_3) & -\sin(\phi_2 - \phi_3) & 0 & A \\
\sin(\phi_2 - \phi_3) & \cos(\phi_2 - \phi_3) & 0 & B \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

where

\[
A = -d \sin(\phi_2 - \phi_3) - (R_2 - R_3) \sin \phi_3 \\
B = d \cos(\phi_2 - \phi_3) - (R_2 - R_3) \cos \phi_3
\]

Here, the matrix \(M_{3, r}\) represents the transformation matrix from coordinate system \((xyz)_r\) to \((xyz)_3\).

In order to express the surface of meshing element, the coordinate system of the cylindrical meshing element is shown in Fig. 3.
The coordinate system attached to the cylindrical meshing element. The height of the cylindrical meshing element of the ring gear is \( t \). The symbol \( \rho \) denotes the radius of the cylindrical meshing element. In coordinate system, the homogeneous coordinate for the position vector of the cylindrical meshing element can be expressed as:

\[
R_r = \begin{bmatrix} \rho \cos \theta & \rho \sin \theta & u & 1 \end{bmatrix}^T
\]

(2)

where \( 0 \leq \theta \leq 2\pi \), \( 0 \leq u \leq t \), and \( R_r \) denotes the surface equation in coordinate system \( r \).

By transforming the equation of the cylindrical meshing element coordinate systems from \((xyz) \) to \((xyz)_r\) [Denavit & Hartenberg, 1955], the surface equation of the cylindrical meshing element can be expressed as:

\[
R'_3 = \begin{bmatrix} C & D & u & 1 \end{bmatrix}^T
\]

(3)

where

\[
C = \rho \sin \theta \cos(\phi_2 - \phi_3) - [d - \rho \cos \theta] \sin(\phi_2 - \phi_3) - (R_2 - R_3) \sin \phi_3
\]

\[
D = \rho \sin \theta \sin(\phi_2 - \phi_3) + [d - \rho \cos \theta] \cos(\phi_2 - \phi_3) - (R_2 - R_3) \cos \phi_3
\]

The gear ratio is defined as \( g = \omega_3 / \omega_2 \), and then Equation (3) can be expressed as:

\[
R'_3 = \begin{bmatrix} E & F & u & 1 \end{bmatrix}^T
\]

(4)

where

\[
E = \rho \sin \theta \cos(1 - g)\phi_2 - (R_2 - R_3) \sin \phi_2 + (\rho \cos \theta - d) \sin(1 - g)\phi_2
\]

\[
F = \rho \sin \theta \sin(1 - g)\phi_2 - (R_2 - R_3) \cos \phi_2 + (d - \rho \cos \theta) \cos(1 - g)\phi_2
\]

Since the cylindrical profile, denoted as \( \Sigma_r \), of the meshing element is a regular surface, it can be expressed in coordinate system \((xyz)_r\), as follows:

\[
R_r(u, \theta) \in C^1, \quad \frac{\partial R_r}{\partial u} \times \frac{\partial R_r}{\partial \theta} \neq 0
\]

(5)

The symbol \( C^1 \) in Equation (5) indicates that the functions \( x(u, \theta) \) and \( y(u, \theta) \) have continuous derivatives to the first order, at least. From the necessary conditions of existence of surfaces [Goetz, 1970; Litvin & Feng, 1996], the equation of meshing can be obtained as follows:

\[
G(u, \theta, \phi_2) = \left( \frac{\partial R'_3}{\partial u} \times \frac{\partial R'_3}{\partial \theta} \right) \cdot \frac{\partial R'_3}{\partial \phi_2} = 0
\]

(6)

Equation (6) relates the curvilinear coordinates \((u, \theta)\) of surface \( \Sigma_r \) with the generalized parameter of motion, \( \phi_2 \). By differentiating Equation (4) with respect to \( u \), \( \theta \), and \( \phi_2 \), respectively, then substituting them into Equation (6), we can obtain the equation of meshing as follows:

\[
\theta = \arctan\left( \frac{R_2 \sin \phi_2}{d - R_2 \cos \phi_2} \right)
\]

(7)

From Equations (4) and (7), the cycloid profiles can be obtained.

**CYCLOIDAL PROFILES AND PINION-TOOTH CENTRES**

Figure 4 shows the geometrical relationships between the cylindrical meshing elements. In order to avoid interference between the near cylindrical meshing elements, the meshing element radius \( \rho \) must be constrained in the following inequality.

\[
\rho < d \sin\left( \frac{\pi}{N_2} \right)
\]

(8)
Because the roller drive is a quasi-cycloid device [Sheu et al., 2001], the proposed reducer having the same behavior with a cycloid drives. Hence, the cylindrical tooth profiles may be used to replace the cycloidal profiles. Solving Simultaneously Equations (4) and (7)-(8), then the cycloidal profiles for cycloid drives with one-tooth difference can be obtained.

From the kinematic gearing principle, for cycloid drives with arbitrary tooth difference, that the relationships of parameters $R_2$, $R_3$, $e$, $N_2$, $N_3$, and $p_c$ can be expressed as follows [Litvin, 1994]:

\[
2\pi R_2 = 2\pi R_3 + (N_2 - N_3)p_c
\]
\[
R_2 = \frac{N_2 e}{N_2 - N_3}
\]
\[
R_3 = \frac{N_3 e}{N_2 - N_3}
\]

where $N_2$ and $N_3$ are the number of the cylindrical meshing elements and pinion teeth, respectively. Symbol $p_c$ denotes the circular pitch. By combining Equations (4) and (7)-(11), cycloidal profiles can be obtained for cycloid drives with arbitrary tooth-difference.

When cycloidal profiles are obtained, the pinion-tooth centre and pinion-tooth radius must be determined subsequently. Figure 5 shows the geometrical relationships, for cycloidal profiles with two-tooth difference, among the cycloidal profiles, pinion-tooth (cylindrical tooth) radius and pinion–tooth centre.

\[
\delta = \frac{360}{N_3}
\]
\[
\overline{ab} = \sqrt{(x_3-x_b)^2 + (y_3-y_b)^2}
\]
\[
\overline{bc} = \rho_3
\]
\[
\overline{ac} = \sqrt{\overline{ab}^2 + \overline{bc}^2 + 2 \times \overline{ab} \times \overline{bc} \times \cos(\gamma)}
\]
\[
R_c = \overline{oa} - \overline{ac}
\]

The cylindrical tooth centre $c$ and cylindrical tooth radius $\rho_3$ can be determined by Equations (12) - (16).

**NUMERICAL EXAMPLES**

In order to design the proposed roller drive, the cycloidal profiles must be obtained first then determined the pinion tooth centres. The major design parameters of the roller drives are shown in Table I. The cycloidal profiles for two-tooth difference are obtained by Equations (4) and (7)-(11). Figures 6-9 show the cycloidal profiles of two-tooth difference for cases 1 – 4, respectively.
TABLE I. Major design parameters and their units in the S.I. system

<table>
<thead>
<tr>
<th>Design parameters</th>
<th>( d )</th>
<th>( e )</th>
<th>( \rho )</th>
<th>( N_2 )</th>
<th>( N_3 )</th>
<th>( R_2 )</th>
<th>( R_3 )</th>
<th>( t )</th>
<th>( \rho_3 )</th>
<th>( R_c )</th>
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<tbody>
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<td>Case 1</td>
<td>60</td>
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<td>8</td>
<td>22</td>
<td>20</td>
<td>22</td>
<td>20</td>
<td>10</td>
<td>6</td>
<td>45.6324</td>
</tr>
<tr>
<td>Case 2</td>
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<td>8</td>
<td>20</td>
<td>18</td>
<td>20</td>
<td>18</td>
<td>10</td>
<td>7</td>
<td>44.6898</td>
</tr>
<tr>
<td>Case 3</td>
<td>60</td>
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<td>7</td>
<td>14</td>
<td>12</td>
<td>14</td>
<td>12</td>
<td>10</td>
<td>10</td>
<td>42.7231</td>
</tr>
<tr>
<td>Case 4</td>
<td>90</td>
<td>4</td>
<td>7</td>
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<td>16</td>
<td>18</td>
<td>16</td>
<td>10</td>
<td>10</td>
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<td>mm</td>
<td>mm</td>
<td>mm</td>
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</tr>
</tbody>
</table>

In order that the cycloidal profiles are replaced by the pinion cylindrical teeth, we choose a suitable cylindrical tooth radius as shown in Table I, the \( R_c \) can be determined by Equations (12)-(16). Figures 10-13 show the drawing of the ring gear and pinion teeth for case 1 and case 2, respectively.
From above results we use CAD (Pro/ENGINEER) constructs the solid modeling for the proposed roller drivers. Figures 14-17 are the solid modeling for cases 1 and 4, respectively.
REFERENCES


Sheu, K. B. Chiu, S. T., Lai, T. S., and Yan, H. S. Quasi-Cycloid Drive Devices, ROC (Taiwan)

CONCLUSIONS

This work presents a concept of new type roller drivers and constructs the topological structure and derives the surface equation for the proposed roller drives. The equation of meshing is derived by envelope theory. Because the roller drive is a quasi-cycloid drive, the cylindrical teeth are used to replace the cycloidal profiles. Using CAD constructs the solid modeling to demonstrate the proposed design procedures and algorithm are useful.

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**NOMENCLATURE**

- $e$ Distance between the centres of the ring gear and the pinion
- $[M_{3,r}]$ Coordinate transformation matrix from system $r$ to system 3
- $R_2$ Radius of the centre of the ring gear
- $R_3$ Radius of the centre of the pinion
- $t$ Height of the ring gear meshing element
- $N_2$ The number of ring-gear meshing elements
- $N_3$ The number of pinion teeth (lobes)
- $R_i$ Surface equation in coordinate system $i$
- $R_j^i$ The $i$ surface equation in coordinate $j$
- $u, \theta$ Curvilinear coordinates on the surface of the cylindrical meshing element
- $(xyz)_2$ Moving coordinate system rigidly connected to the ring gear
- $(xyz)_3$ Moving coordinate system rigidly connected to the pinion
- $(xyz)_b$ Fixed coordinate system rigidly connected to the frame at point $b$
- $(xyz)_f$ Fixed coordinate system rigidly connected to the frame at point $f$
- $(xyz)_r$ Moving coordinate system rigidly connected to the cylindrical meshing element
- $\phi_2$ Angular displacement of the ring gear
- $\phi_3$ Angular displacement of the pinion
- $\Sigma_2$ Ring gear surface
- $\Sigma_r$ Cylindrical meshing element
- $\rho$ Radius of cylindrical meshing element
- $\rho_3$ Radius of pinion teeth